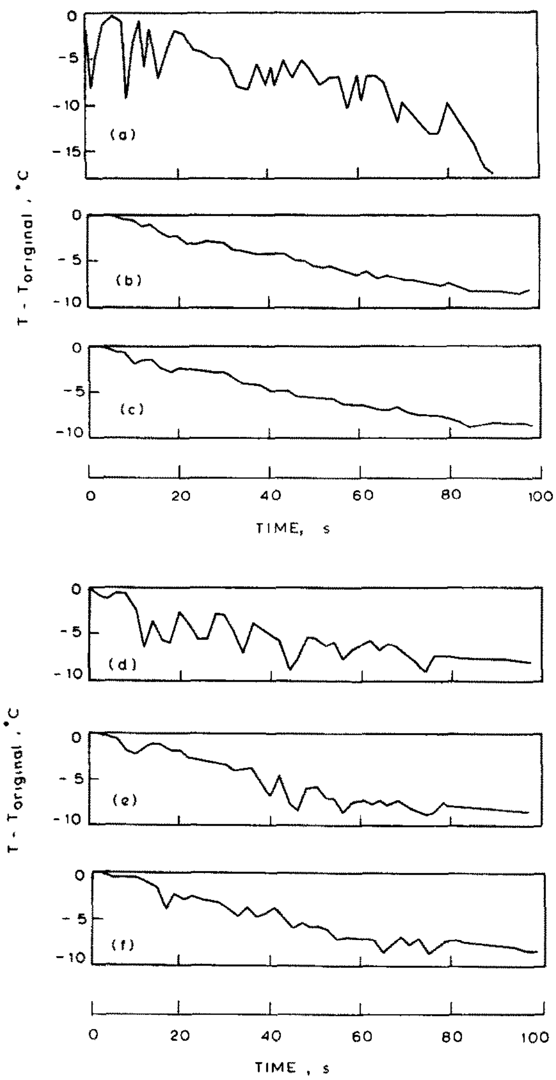

FIG. 2. Water temperatures after a spill of liquid nitrogen on 42.5°C water. 0.95 g/cm^2 of nitrogen was spilled. Thermocouple locations measured below the interface were as follows: (a) 0.3 cm, center; (b) 5.4 cm, center; (c) 3.1 cm, center; (d) 1.3 cm, side; (e) 3.4 cm, side; (f) 4.1 cm, side.

descends and mixes surprisingly thoroughly with the bulk—at least to depths of 6–6.5 cm as employed in these tests. The fluctuations noted in Fig. 2 attest to these thermals.

Although the energy transfer process in the water phase appears to depend upon the initial water temperature, the actual boil-off rate of cryogen is not affected. These findings cast some doubt on earlier theories [1, 3] which postulate that, for liquid methane, the boil-off increases with time due to ice formation which encourages a change in the boiling regime from film (on liquid water) to nucleate (on surface ice).

It is now established [1] that the boiling rate of liquid nitrogen on water decreases with time whereas, as noted above, for liquid methane the rate increases. But, the experiments reported herein indicate that there is no significant difference in the temperature response of the water between nitrogen and methane spills if the initial water temperatures are the same.

It is also interesting to note that even in the case of cool water with a growing ice film, it was not possible to correlate the heat transfer rate with theory using a conduction model with a moving ice boundary. The conduction model overestimates the boil-off rate and also predicts a time dependence different from that observed experimentally.

In general, the mechanism by which the water phase supplies energy to evaporate the cryogen is dependent on initial water temperature. At higher temperatures, the energy is supplied primarily by convection and homogeneous cooling of water. For low water temperatures, most of the energy is supplied by the heat of formation of ice. However, the change in the mechanism of heat transfer in the water phase was not found to affect the transient boiling rate for either liquid methane or nitrogen on water. These conclusions pertain only to early transient boiling phenomena—during about the first 100 s—after a spill. At longer times than those studied, ice buildup would be expected to limit and decrease boiling rates.

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LAMINAR HYPERSONIC BOUNDARY-LAYER FLOW AT A THREE-DIMENSIONAL STAGNATION POINT WITH SLIP AND MASS TRANSFER

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NOMENCLATURE

a, b, velocity gradients in x and y directions respectively;
c, ratio of velocity gradients, b/a;
C_{fx}, C_{fy}, skin-friction coefficients along x and y directions respectively;
f, F, dimensionless stream functions such that $f' = u/u_e$ and $F' = v/v_e$;
f_w, mass-transfer parameter, $-(\rho w)_w/(\rho_e \mu_e a)^{1/2}$;
g, dimensionless enthalpy, h/h_e;
g_w, cooling parameter for the wall, h_w/h_e;
$g(0)$, cooling parameter for the gas defined by (2c);

h, enthalpy;
Kn, Knudsen number;
l, molecular mean free path of the gas;
q, heat-transfer rate;
Re_x, local Reynolds number;
St, Stanton number;
T, temperature;
u, v, w, velocity components along x, y, z directions respectively;
x, y, z, principal, transverse and normal directions respectively.

Greek symbols
α, α_1, reflection and thermal accommodation coefficients respectively;
γ, ratio of specific heats;
η, similarity variable, $(\rho_e a/\mu_e)^{1/2} \int_0^z (\rho/\rho_e)\,dz$;
λ, λ_1, slip parameters defined by (3);
μ, coefficient of viscosity;
ν, kinematic viscosity;
ρ, density,
σ, Prandtl number;
τ_x, τ_y, dimensional shear stress functions;
ω, exponent in the power-law variation of viscosity.

Superscript
$'$, denotes differentiation with respect to η.

Subscripts
e, denotes condition at the edge of the boundary layer;
w, denotes condition at the surface $z = \eta = 0$.

INTRODUCTION

THE BOUNDARY layer greatly influences the performance characteristics of high-altitude flight vehicles which move at high Mach number in the rarefied atmosphere. An accurate prediction of the hypersonic boundary-layer flow-field and the heat transfer at a general three-dimensional stagnation point for rarefied gases is essential for the design of flight vehicles such as space vehicles and re-entry satellites. The laminar compressible three-dimensional stagnation point flow of a gas with or without mass transfer was studied by Poots [1], Libby [2], Wortman *et al.* [3] and Vimala and Nath [4]. However, the effect of slip was not considered by them.

In this paper, the hypersonic flow of a viscous slightly rarefied gas (i.e. slip-flow regime, where, $0.01 < Kn < 0.1$; see [5]) with variable properties near a three-dimensional stagnation point with mass transfer has been investigated. The continuum approach has been followed but the appropriate slip velocities and temperature jump boundary conditions have been used in place of no-slip conditions. The experimental and theoretical results in rarefied gas dynamics seem to indicate that this approach is not only adequate but probably superior to other existing techniques [5].

GOVERNING EQUATIONS

The governing equations in dimensionless form for steady laminar hypersonic flow of a slightly rarefied gas with variable properties (i.e. $\rho \propto T^{-1}$, $\mu \propto T^{\omega}$, $\sigma = 0.7$) in the neighbourhood of the stagnation point of a three-dimensional porous body are [2–4]

$$f''' + (\omega-1)g'f''/g + [(f+cF)f'' + g - f'^2]g^{1-\omega} = 0 \quad (1a)$$

$$F''' + (\omega-1)g'F''/g + [(f+cF)F'' + c(g-F'^2)]g^{1-\omega} = 0 \quad (1b)$$

$$g'' + (\omega-1)g'^2/g + \sigma(f+cF)g'g^{1-\omega} = 0. \quad (1c)$$

The appropriate boundary conditions to account for the first-order slip velocity and temperature jump are [5–6]

$$f(0) = f_w,\ f'(0) = \lambda f''(0),\ f'(\infty) \to 1 \quad (2a)$$

$$F(0) = 0,\ F'(0) = \lambda F''(0),\ F'(\infty) \to 1 \quad (2b)$$

$$g(0) = g_w + \lambda_1 g'(0),\ g(\infty) \to 1. \quad (2c)$$

Here $f_w \gtrless 0$ according to whether there is suction or injection. The slip parameters λ and λ_1 are defined by [6]

$$\lambda = [(2-\alpha)/\alpha][l\rho(a/\rho_e\mu_e)^{1/2}] \quad (3a)$$

$$\lambda_1 = [(2-\alpha_1)/\alpha_1][2\gamma l/(\gamma+1)\sigma][\rho(a/\rho_e\mu_e)^{1/2}]. \quad (3b)$$

It has been found [6] that λ and λ_1 are of the same order of magnitude, hence we have taken $\lambda \sim \lambda_1$. The value of λ is small compared to unity and it is zero for no-slip flow.

It may be noted that $\omega = 0.5$ corresponds to the conditions encountered in hypersonic flight, $\omega = 0.7$ corresponds to low-temperature flows and $\omega = 1$ represents the constant density–viscosity product simplification [7]. It is to be mentioned that most shapes of practical interest range from sphere ($c = 1$) to cylinder ($c = 0$) and the saddle shapes ($-1 \leq c < 0$) are included in the analysis for the sake of completeness [2].

The skin-friction coefficients along x and y directions are given by [2]

$$C_{fx} = 2\tau_x/\rho_e u_e^2 = 2(Re_x)^{-1/2} f_1''(0) \quad (4a)$$

$$C_{fy} = 2\tau_y/\rho_e u_e^2 = 2(Re_x)^{-1/2}(v_e/u_e)F_1''(0). \quad (4b)$$

Similarly, the heat-transfer coefficient in terms of Stanton number can be expressed as [2]

$$St = q_w/[(h_e - h_w)\rho_e u_e] = (Re_x)^{-1/2} G'(0) \quad (5)$$

where

$$f_1''(0) = g^{\omega-1}(0)f''(0),\ F_1''(0) = g^{\omega-1}(0)F''(0) \quad (6a)$$

$$G'(0) = \sigma^{-1}g^{\omega-1}(0)g'(0)/(1-g_w),\ Re_x = u_e x/\nu_e. \quad (6b)$$

It may be noted that for no-slip flow ($\lambda = 0$), $g(0) = g_w$, but for slip flow ($\lambda > 0$), $g(0) \neq g_w$ and the relation between them is governed by equation (2c).

RESULTS AND DISCUSSION

The governing equations (1) under conditions (2) have been solved numerically by the method of parametric differentiation [8–11] for various values of λ, ω, c, f_w and g_w taking $\sigma = 0.7$. The starting solutions for carrying out computations in the present case have been obtained by using the wall values for $\omega = 1$, $\lambda = c = f_w = g_w = 0$ tabulated in [2].

Numerical computations were carried out for 288 conditions involving several parameters. However, for the sake of brevity, only some representative velocity, enthalpy, shear-stress and heat-transfer profiles are displayed in Figs. 1–3. The effect of the slip parameter λ or the injection parameter $f_w(f_w < 0)$ or $\omega(\omega < 1)$ is to make velocity and enthalpy profiles less steep, whereas they become more steep when the suction parameter $f_w(f_w > 0)$ or the cooling parameter g_w increases (the profiles for $g_w = 0.6$, $f_w \neq 0$ and $\omega = 0.7$ are not presented in figures for lack of space). The velocity and enthalpy profiles (f', F', g) have a point of inflexion for $f_w \geq 0$ and $g_w = 0.2$ when $\omega = 0.5$ (or 0.7) as is evident from

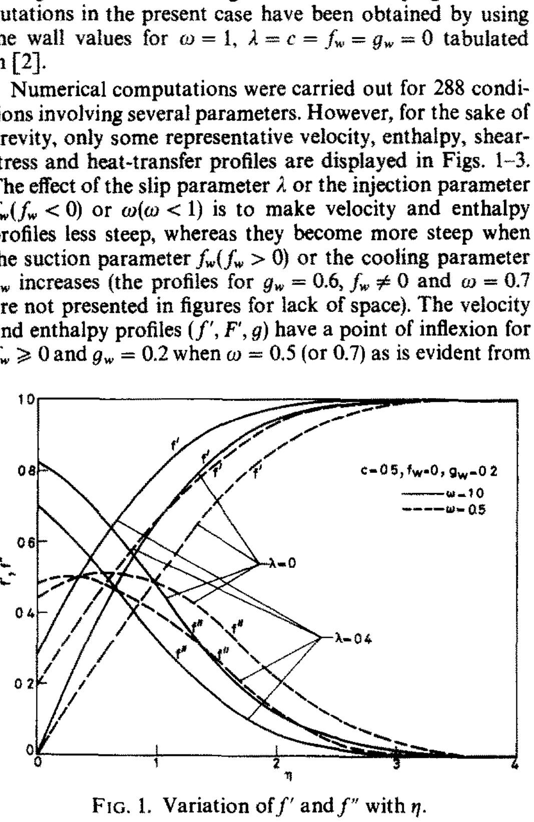

FIG. 1. Variation of f' and f'' with η.

Table 1. Skin-friction and heat-transfer parameters ($c = 0.5$, $g_w = 0.2$) $\sigma = 0.7$

		$\omega = 1.0$			$\omega = 0.7$			$\omega = 0.5$		
f_w	λ	$f_1''(0)$	$F_1''(0)$	$G'(0)$	$f_1''(0)$	$F_1''(0)$	$G'(0)$	$f_1''(0)$	$F_1''(0)$	$G'(0)$
0	0	0.8208	0.7297	0.7691	0.9241	0.8142	0.8380	1.0062	0.8803	0.8926
	0.2	0.7653	0.6943	0.7535	0.8697	0.7807	0.8253	0.9529	0.8485	0.8814
	0.4	0.6955	0.6419	0.7175	0.7970	0.7279	0.7921	0.8798	0.7969	0.8510
	0.6	0.6277	0.5869	0.6735	0.7205	0.6676	0.7467	0.7981	0.7339	0.8060
0.5	0	1.1725	1.0867	1.1403	1.2638	1.1613	1.1994	1.3375	1.2215	1.2505
	0.2	1.0116	0.9517	1.0458	1.1273	1.0511	1.1273	1.2163	1.1265	1.1916
	0.4	0.8675	0.8258	0.9432	0.9812	0.9269	1.0319	1.0724	1.0067	1.1035
	0.6	0.7513	0.7214	0.8492	0.8506	0.8117	0.9335	0.9330	0.8860	1.0041
1.0	0	1.5604	1.4822	1.5435	1.6396	1.5469	1.5933	1.7051	1.6003	1.6389
	0.2	1.2471	1.1982	1.3301	1.3827	1.3197	1.4301	1.4826	1.4075	1.5050
	0.4	1.0177	0.9862	1.1462	1.1460	1.1047	1.2526	1.2470	1.1968	1.3371
	0.6	0.8534	0.8319	0.9992	0.9592	0.9315	1.0950	1.0449	1.0114	1.1730
−0.5	0	0.5199	0.4285	0.4451	0.6333	0.5202	0.5233	0.7220	0.5918	0.5860
	0.2	0.5211	0.4423	0.4678	0.6239	0.5253	0.5387	0.7069	0.5922	0.5971
	0.4	0.5058	0.4404	0.4757	0.6012	0.5182	0.5433	0.6796	0.5818	0.5994
	0.6	0.4812	0.4278	0.4723	0.5693	0.5009	0.5373	0.6428	0.5614	0.5919

Table 2. Skin-friction and heat-transfer parameters ($c = -0.5$, $g_w = 0.2$) $\sigma = 0.7$

		$\omega = 1$			$\omega = 0.7$			$\omega = 0.5$		
f_w	λ	$f_1''(0)$	$F_1''(0)$	$G'(0)$	$f_1''(0)$	$F_1''(0)$	$G'(0)$	$f_1''(0)$	$F_1''(0)$	$G'(0)$
0	0	0.7072	0.1535	0.5768	0.8075	0.1758	0.6403	0.8879	0.1965	0.6894
	0.2	0.6584	0.1606	0.5710	0.7585	0.1819	0.6303	0.8389	0.2009	0.6782
	0.4	0.6030	0.1596	0.5510	0.7004	0.1815	0.6108	0.7797	0.2005	0.6591
	0.6	0.5501	0.1541	0.5260	0.6407	0.1764	0.5855	0.7165	0.1955	0.6335
0.5	0	1.0292	0.5301	0.9058	1.1154	0.5386	0.9519	1.1861	0.5469	0.9941
	0.2	0.8903	0.4889	0.8358	0.9978	0.5129	0.8989	1.0811	0.5292	0.9496
	0.4	0.7714	0.4450	0.7651	0.8792	0.4772	0.8353	0.9656	0.4998	0.8917
	0.6	0.6757	0.4045	0.7016	0.7734	0.4388	0.7701	0.8547	0.4644	0.8276
1.0	0	1.4047	1.0182	1.2912	1.4757	1.0110	1.3241	1.5356	1.0077	1.3571
	0.2	1.1282	0.8559	1.1216	1.2519	0.8989	1.1973	1.3431	0.9235	1.2541
	0.4	0.9309	0.7321	0.9814	1.0528	0.7885	1.0675	1.1489	0.8267	1.1351
	0.6	0.7887	0.6377	0.8685	0.8937	0.6927	0.9507	0.9798	0.7330	1.0175
−0.5	0	0.4461	−0.0386	0.3216	0.5570	−0.0231	0.3919	0.6446	−0.0053	0.4510
	0.2	0.4432	−0.0308	0.3367	0.5446	−0.0168	0.4001	0.6269	−0.0010	0.4544
	0.4	0.4305	−0.0243	0.3450	0.5246	−0.0113	0.4035	0.6022	0.0023	0.4541
	0.6	0.4121	−0.0195	0.3473	0.4995	−0.0073	0.4025	0.5724	0.0051	0.4503

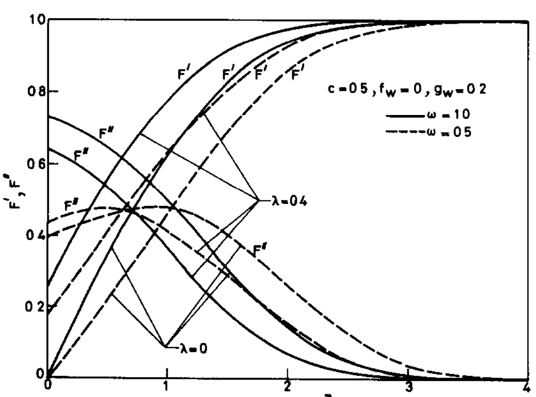

FIG. 2. Variation of F' and F'' with η.

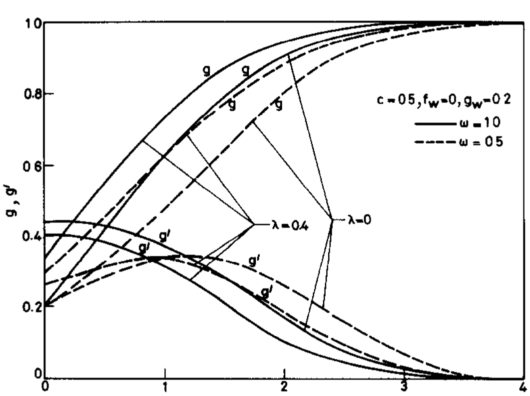

FIG. 3. Variation of g and g' with η.

a maximum in $f''(\eta)$, $F''(\eta)$ and $g'(\eta)$ shown in Figs. 1–3 but there is no point of inflexion when $\omega = 1$. But in the case of injection, it is present even for $\omega = 1$. For $g_w = 0.6$, the point of inflexion occurs only in the case of enthalpy profiles. These results hold good both for slip and no-slip flows. Similar effects have been observed by Gross and Dewey [7] for two-dimensional stagnation-point flow without slip. It may be noted that the existence of a point of inflexion implies that the flow is prone to instability [12]. Gross and Dewey [7] have mentioned that the occurrence of a point of inflexion in the velocity profiles when $\omega \neq 1$ may explain the anomalous result of stability theories that the critical transition Reynolds number is infinite for a highly cooled flat plate when $\omega = 1$.

For the sake of brevity, the skin-friction and heat-transfer parameters, $f_1''(0)$, $F_1''(0)$ and $G'(0)$ only for $c = 0.5$, -0.5 and $g_w = 0.2$ are given in Tables* 1–2. We observe that at both saddle ($c < 0$) and nodal ($c \geqslant 0$) points of attachment, as suction is increased, the effect of slip on $f_1''(0)$, $F_1''(0)$ and $G'(0)$ is more pronounced and they [$f_1''(0)$, $F_1''(0)$ and $G'(0)$] decrease as λ increases. This is independent of the exponent ω. But, for a fixed value of λ, $g_w(0 < g_w < 1)$ and f_w, we find that $f_1''(0)$, $F_1''(0)$ and $G'(0)$ increase as ω decreases. This trend holds for $c \gtrless 0$. It may be appropriate to mention that for a given $g_w(0 < g_w < 1)$ the parameters $f''(0)$, $F''(0)$ and $g'(0)$ which occur in skin-friction and heat-transfer coefficients [see equations (4) to (6)] decrease as ω decreases, but $g^{\omega-1}(0)$ increases as ω decreases. Consequently, as mentioned earlier, $f_1''(0)$, $F_1''(0)$ and $G'(0)$ [see equations (6)] increase as ω decreases. It is seen that for all values of λ, f_w and ω, $f_1''(0)$, $F_1''(0)$ and $G'(0)$ decrease as c decreases until at some negative c, $F_1''(0)$ [i.e. $F''(0)$] is reversed and $f_1''(0)$ and $G'(0)$ begin to increase as c decreases. This trend has also been observed by Libby [2] and Wortman *et al.* [3]. It is also observed that $f_1''(0)$, $F_1''(0)$ and $G'(0)$ are increased due to suction or due to increase in g_w and the effect of injection is just the reverse. We have compared our heat-transfer results [$G'(0)$] for $\lambda = f_w = 0$; $g_w = 0.2$, 0.6 and $\omega = 1$ with those tabulated by Wortman *et al.* [3] and they are found to be in excellent agreement except when $c = -1$ in which case the difference is about 1.5%. The skin-friction results could not be compared with those of [3] as they were not available in tabular form.

*The results for $c = 0, 1, -1$ (when $g_w = 0.2$ or 0.6) and $c = 0.5, -0.5$ (when $g_w = 0.6$) can be supplied to the reader on request.

CONCLUSIONS

The effect of slip on skin friction and heat transfer is more pronounced when suction is increased. The skin friction and heat transfer are strongly dependent on the nature of the stagnation point and they are increased due to suction, but the effect of injection is just the reverse. The effect of the variation of the density–viscosity product across the boundary layer is to increase skin friction and heat transfer and this variation gives rise to a point of inflexion in velocity and enthalpy profiles.

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